KILROY ACADEMY

NAME: ____________________ **DATE:** ____________

WHAT ARE TIDES?

Complete the following as you watch the video *What are Tides?*

Answer the following questions as you watch the video, "What are Tides?"

1. What are tides?

   ____

   ____

2. What are two things you need to do in order to predict the tides?

   - ____

     ____

   - ____

     ____

3. The ____________________ ____________________ is created from tracing all of the possible right triangles with a hypotenuse equal to one.

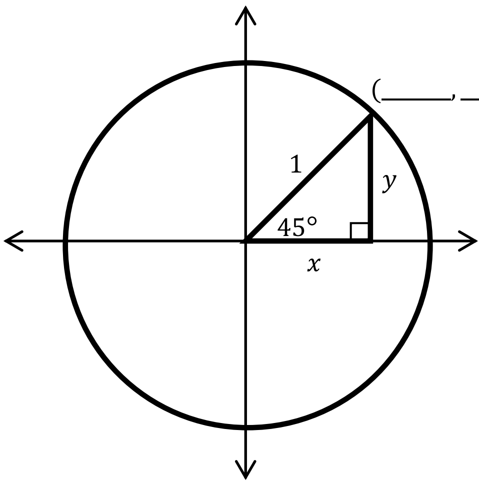
4. The cosine of the angle results in the ____________________ coordinate and the sine of the angle results in the ____________________ coordinate.

5. If you move completely around the circle once in a second and plot the y-values against time, a wave that looks like a tidal wave is created. Each wave is a simple ____________________ wave.

REVIEWING RIGHT TRIANGLE TRIGONOMETRY

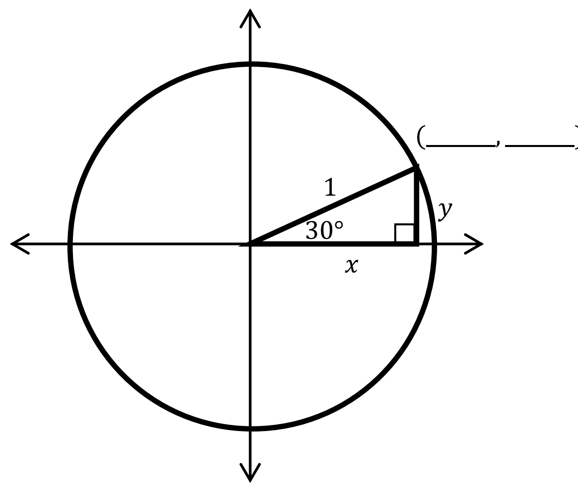
45°-45°-90°

Use the Pythagorean Theorem to find x and y. *Remember that x and y are the same!*



30°-60°-90°

Use the 30°-60°-90° shortcut rules to find x and y.
Remember, this triangle is half of an equilateral triangle!

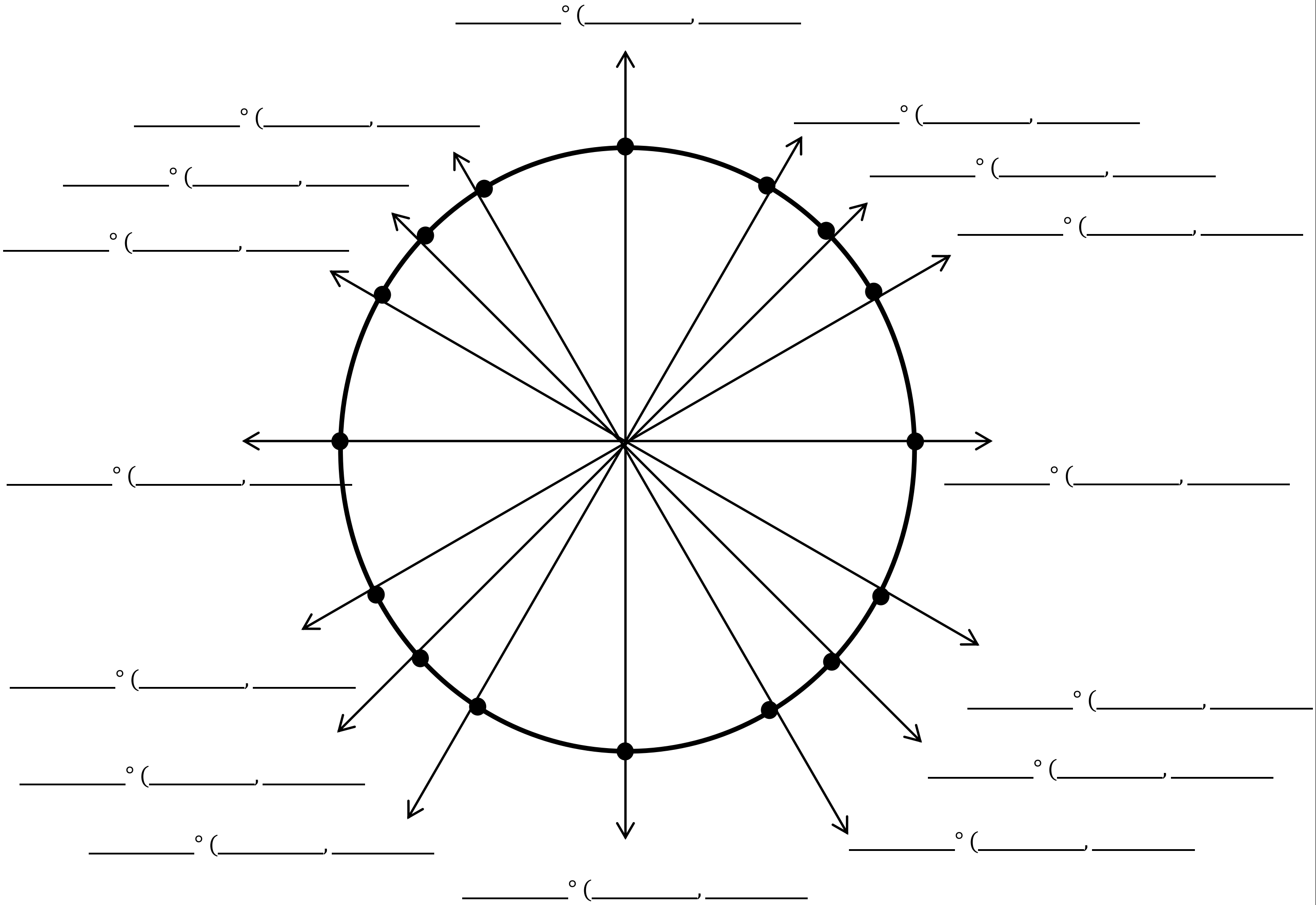


How could you adapt your findings for the 30° angle shown for a 60° angle created at the origin instead?

Draw in the new triangle above and label the coordinate point on the unit circle.

THE UNIT CIRCLE

Use the coordinates we derived from the special right triangles, the key points on the x- and y-axes, and reflections to complete the unit circle below. Remember that one rotation around the circle is 360°.



Reflect on the patterns you see in the coordinates of the unit circle. Describe your observation below. Also, tell how these observations can help you to remember these important coordinate points.

RADIAN MEASURE

Angles can also be measured in **radians**. One radian is the size of the central angle subtended by an arc which is the same length as the radius of the circle.

One complete rotation around a circle is subtended by an arc equal to the circumference of the circle or $2\pi r$. Since the unit circle has a radius equal to one, it follows that $2\pi = 360°$.

Therefore π radians = $180°$ and the conversion factors follow: **1 radian =** $\frac{\mathbf{180}}{\boldsymbol{\pi}}$ **degrees** and **1 degree** = $\frac{\boldsymbol{\pi}}{\mathbf{180}}$ **radians**.

To convert angle measures into different units, simply multiply by the conversion factor.

Convert the following angle measures, given in degrees, to radians.

1. $30°$
2. $120°$
3. $270°$
4. $60°$
5. $45°$
6. $315°$
7. $180°$
8. 225

Convert the following angle measures, given in radians, to degrees.

9. $\frac{\pi}{3}$
10. $\frac{3\pi}{2}$
11. π
12. $\frac{5\pi}{4}$
13. $\frac{2\pi}{3}$
14. $\frac{7\pi}{6}$
15. $\frac{\pi}{2}$
16. $\frac{11\pi}{6}$

Go back to your unit circle and label all of the angles with their corresponding radian measure.

GRAPHING THE SINE WAVE

The quadrantles are the angles where the unit circle intersects the x- and y-axes.

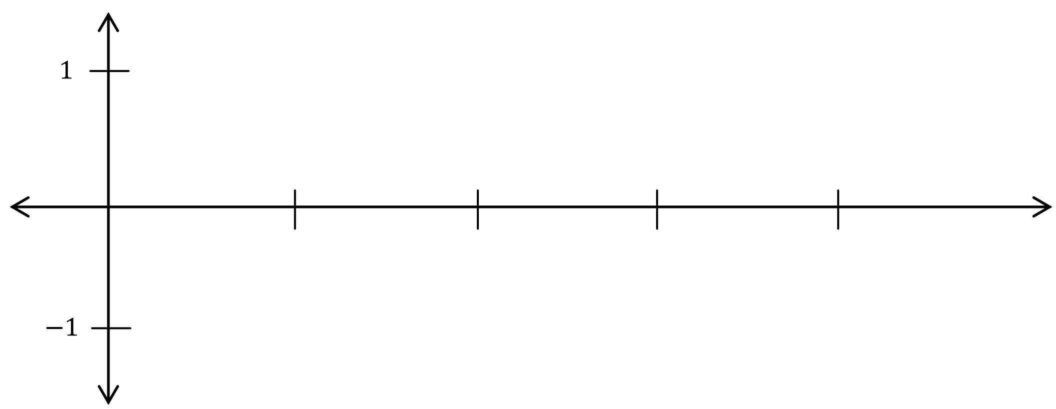
Write down the quadrantles, in radian measure. ____________, ____________, ____________,

____________, ____________.

What observations can you make regarding the coordinate points at these angles?

On the graph below, use the quadrantles and plot their sine values against the angle measure for one full rotation about the unit circle.

$$\boldsymbol{y = \sin x}$$



This function is the parent function for all sinusoidal waves. The function can be transformed to create other sine waves with different amplitudes, periods, and shifts.

$$\boldsymbol{y = a\sin(bx + c) + d}$$

Key Features for the Sine Function

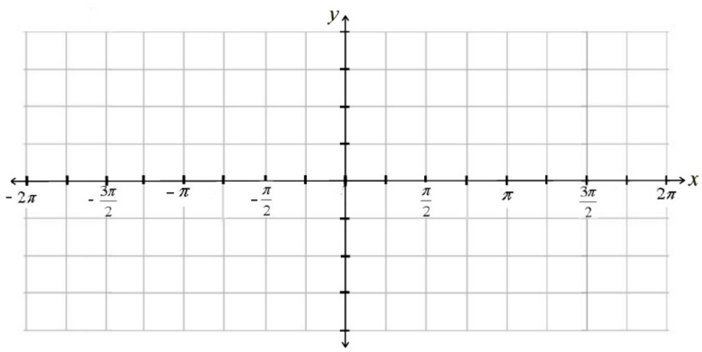
- **Amplitude** is the height of the sine wave from the principal axis, given by $|a|$.
- The sine function is periodic, that is, it repeats the same cycle of values over and over. One **period** is defined as the length of one cycle and is determined by b. Period = $\frac{2\pi}{b}$
- Shifting the function left or right is known as a **phase shift**. The parent function starts at $(0, 0)$. If you set $bx + c = 0$ and solve for x, you will find the beginning x-value for any sine wave.
- A **vertical shift** is represented by d. The entire wave is shifted up or down this many units.

1. What is the value of a, b, c, and d for the function $y = \sin x$? Explain what this tells you about the function.
2. What is the principal axis?
3. What is the period?
4. Explain what would happen to the graph if it were $y = -\sin x$? Sketch and label it on the above graph.

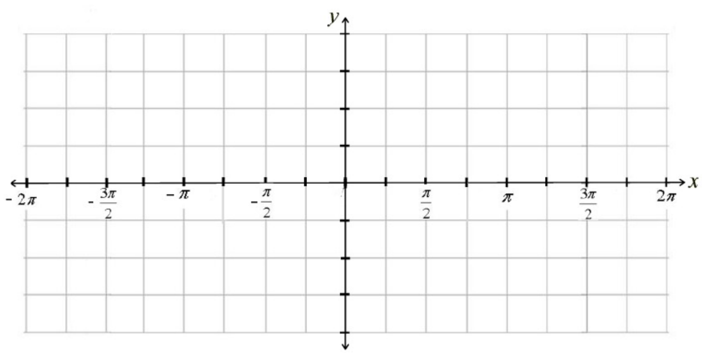
GRAPHING SINUSOIDAL FUNCTIONS

Graph the following sine functions. State the amplitude, period, phase shift, and vertical shift.

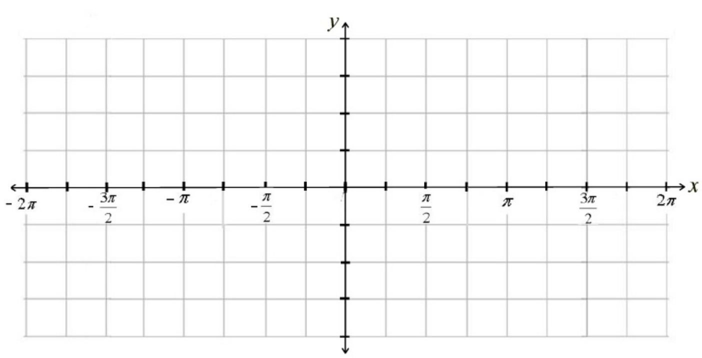
1. $y = \sin x - 1$ Amplitude: ________ Period: ________ Phase Shift: ________ Vertical Shift: ________



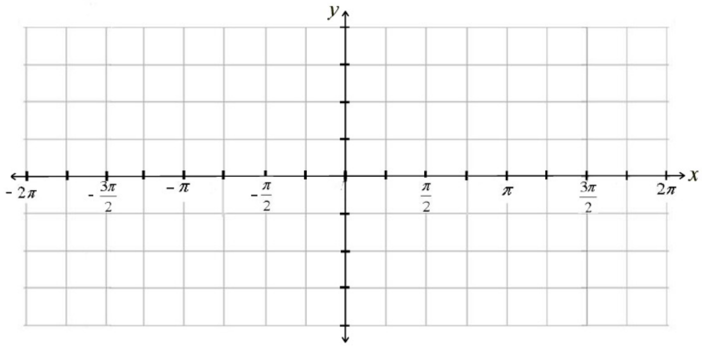
2. $y = \sin x + 2$ Amplitude: ________ Period: ________ Phase Shift: ________ Vertical Shift: ________



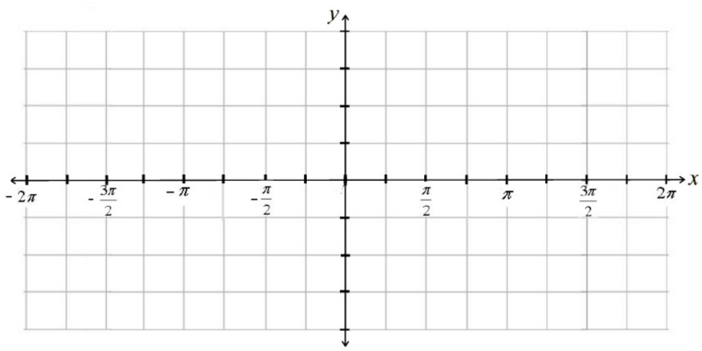
3. $y = 2\sin(x)$ Amplitude: ________ Period: ________ Phase Shift: ________ Vertical Shift: ________



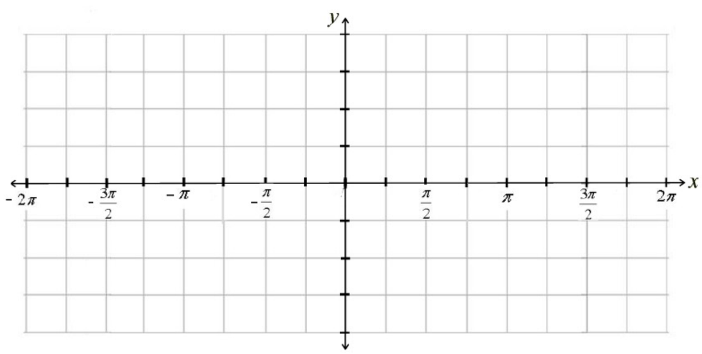
4. $y = -3\sin(x)$ Amplitude: ________ Period: ________ Phase Shift: ________ Vertical Shift: ________



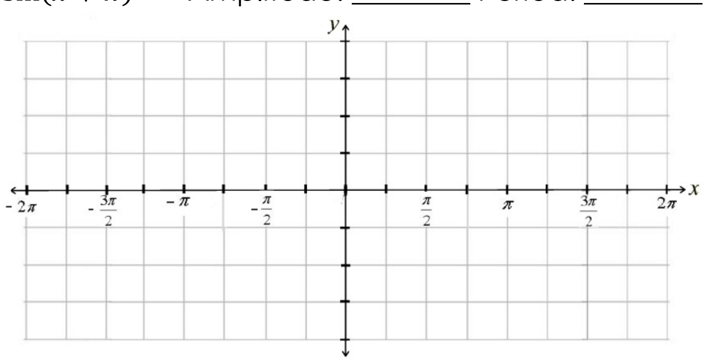
5. $y = \sin(2x)$ Amplitude: ________ Period: ________ Phase Shift: ________ Vertical Shift: ________



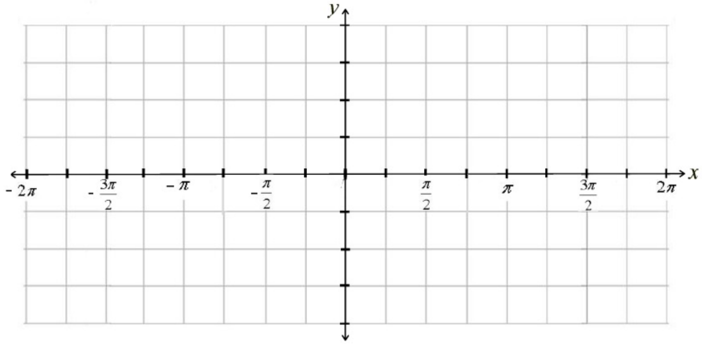
6. $y = \sin\left(\frac{1}{2}x\right)$ Amplitude: ________ Period: ________ Phase Shift: ________ Vertical Shift: ________



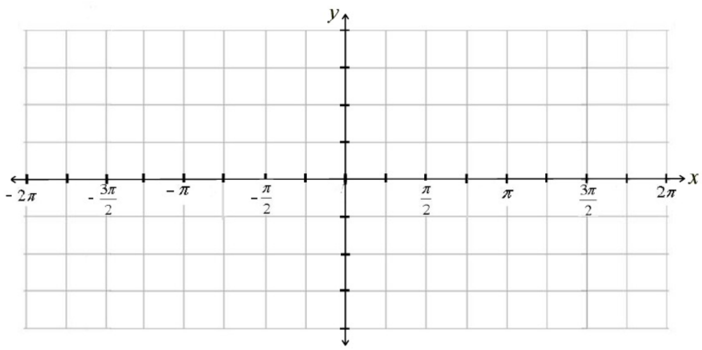
7. $y = \sin(x + \pi)$ Amplitude: ________ Period: ________ Phase Shift: ________ Vertical Shift: ________



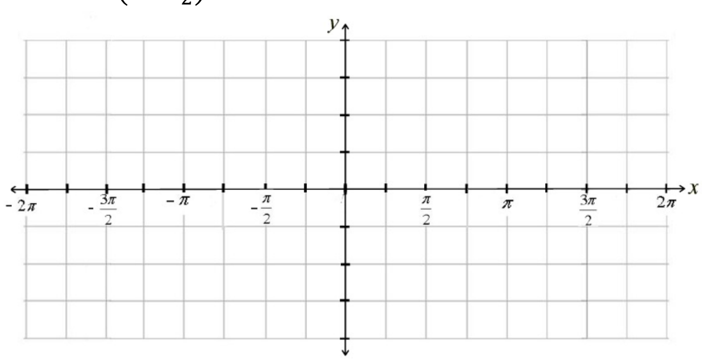
8. $y = \sin\left(x - \frac{\pi}{2}\right)$ Amplitude: ________ Period: ________ Phase Shift: ________ Vertical Shift: ________



9. $\sin\left(x + \frac{\pi}{2}\right) - 2$ Amplitude: ________ Period: ________ Phase Shift: ________ Vertical Shift: ________

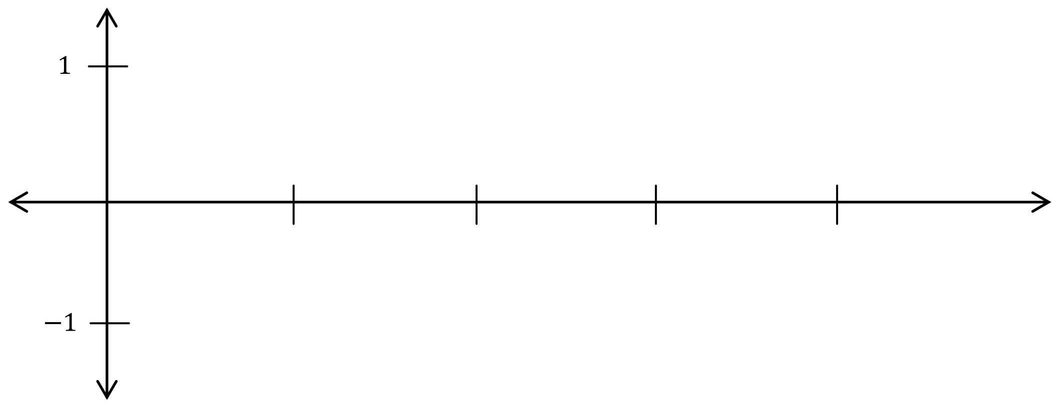


10. $y = 2\sin\left(x - \frac{\pi}{2}\right)$ Amplitude: ________ Period: ________ Phase Shift: ________ Vertical Shift: ________



EXTENSION TO GRAPH OF COSINE

Look back to the unit circle and your comparisons of the sine values for angles and the cosine values for the angles. There is a noticeable pattern that was described. Explain how the observed pattern you described would apply to change the graph of the function $y = \sin x$ into the graph of the function $y = \cos x$. Make a quick sketch of what you think $y = \cos x$ should look like.



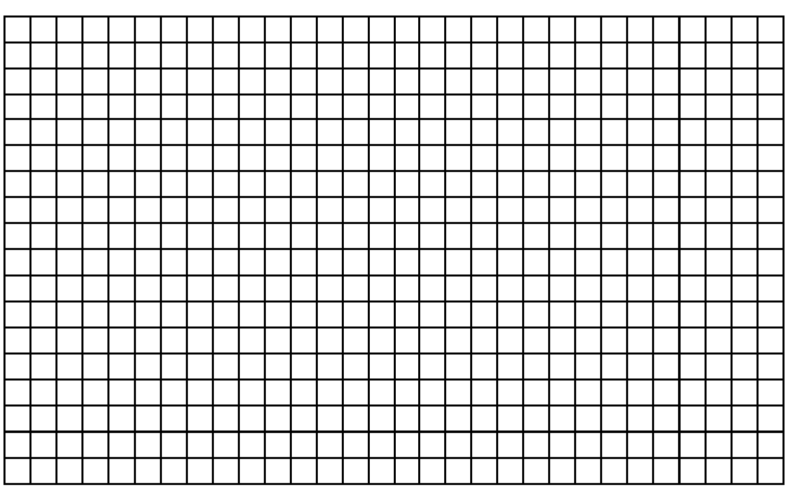
APPLICATIONS OF SINE TO TIDES

1. A group of Kilroy Academy students from Fort Pierce, Florida, decided to study the sinusoidal nature of tides. Values for the depth of the water level were recorded at various times.

At $t = 2$ hours low tide was recorded at a depth of 3 ft or 0.9144 m.

At $t = 8.2$ hours, high tide was recorded at a depth of 1.8 m.

(a) Sketch the graph of this function.
Be sure to label the axes and use appropriate scales.



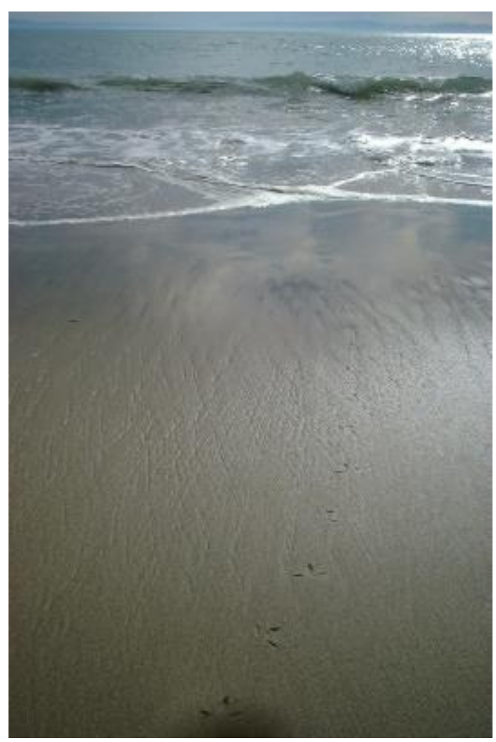


Photo from Ryan Brown
http://www.tides.info/?command=view&location=Fort%20Pierce%20Inlet%2C%20Florida

(b) Write the equation of the sinusoidal function expressing depth in terms of time.

(c) Find the depth of water at $t = 21$ hours.

2. Collect data from a Kilroy water monitor by visiting http://api.kilroydata.org/public/. Use two of the most recent times and depths for a consecutive high tide and low tide. Complete the following table with your findings.

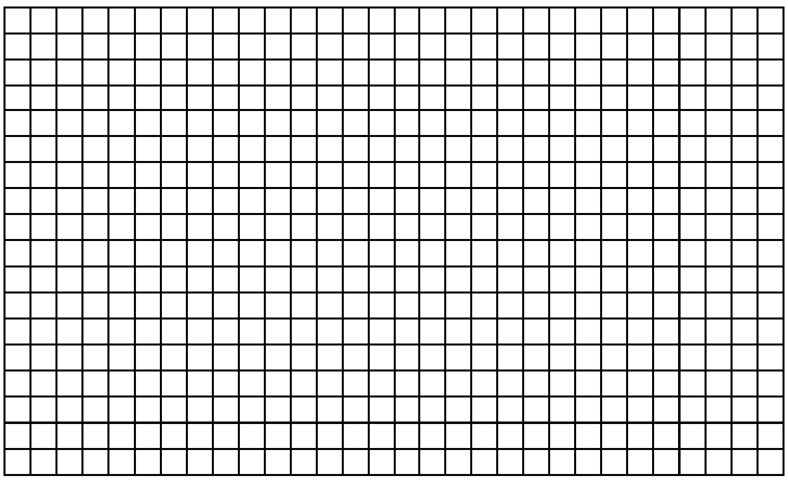
Kilroy Location:		
Date	**Time**	**Water Depth**

Using the data from your table above, sketch a graph of the water depth as a function of time. Set the first time in your table to $t = 0$ hours.

Using the information from the graph and or table, write the equation for the sinusoidal function that can be used to represent the tides at this Kilroy location.

3. Choose a Kilroy™ monitoring device in a different location and repeat the same process used in #2.

Kilroy Location:		
Date	**Time**	**Water Depth**



Function:

ACKNOWLEDGEMENTS

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